**Error Analysis Training**

This training has two experiments. 1) acceleration due to gravity by free fall, for which we use two different setups and 2) volume of metal bar

## Measurement of acceleration due to gravity 'g' by free fall

SETUP-1 with one detector for measuring time:

Equation to find 'g' is $g = \frac{2h}{t^2}$ ----(1)

Measure the time 't' of free fall of given ball at large number of times $t_1$ , $t_2$ , ........$t_n$ at a fixed heights 'h'. Calculate corresponding mean time '$\bar{t}$', error in t. Measure 'h' in best possible way and write down error in 'h' by estimating with least count of meter scale/ scale attached to the setup.

| **Height (h)= …cm** | |
|---|---|
| **Sl No.** | **Time in sec** |
| 1 | |
| 2 | |
| … | |
| …. | |
| 30 | |
| $\bar{t}$ | |

• determine 'g' and error in 'g',

$$\delta g = \sqrt{\left(\frac{\partial g}{\partial h}\,\delta h\right)^2 + \left(\frac{\partial g}{\partial t}\,\delta t\right)^2} = \sqrt{\left(\frac{2}{t^2}\delta h\right)^2 + \left(\frac{4h}{t^3}\,\delta t\right)^2}$$

• Report the 'g' value with errors, is there any systematic error? and if so, explain.

$$g = g_{\text{measured}} \pm \delta g$$

<u>SETUP-2 with two detectors (detector 'a' and detector 'b') for measuring time:</u>

Equation to find 'g' is $g = \frac{2(h_b - h_a)}{(t_b^2 - t_a^2)}$ ----(2)

Measure the time '$t_a$' and '$t_b$' of free fall of given ball at large number of times $t_{1a}$ , $t_{1b}$, $t_{2a}$, $t_{2b}$ ........$t_{na}$, $t_{nb}$ at a fixed heights '$h_a$ and $h_b$ for two detectors'. Calculate corresponding '$\bar{t}_a, \bar{t}_b$,', error in $t_a$ and $t_b$. Measure '$h_b$ and $h_a$' in best possible way and write down error in '$h_b$' and '$h_a$' by estimating with least count of meter scale/ scale attached to the setup.

| **Heights ($h_a$)= …cm, $h_b$= …cm** | | |
|---|---|---|
| **Sl No.** | **Time $t_a$ in sec** | **Time $t_b$ in sec** |
| 1 | | |
| 2 | | |
| … | | |
| …. | | |
| 30 | | |
| | $\bar{t}_a$= …. | $\bar{t}_b$=… |

To determine in error in 'g', error in $h_a$, $h_b$, $t_a$ and $t_b$ need to be propagated.

To simplify calculation of error in 'g', we use the following.

Equation 2 can be written as $g = \frac{2(h_{difference})}{T^2_{difference}}$

where $h_{difference}$ ($h_d$) is ($h_b$-$h_a$) and $T^2_{difference}$ ($T_d^2$) i.e. ($t_b^2$- $t_a^2$)

Error in $h_d$ is $\delta h_{difference}(\delta h_d) = \sqrt{(\delta h_b^2 + \delta h_a^2)}$

Error in $T_{difference}$ is $\delta T_{difference}(\delta T_d) = \sqrt{(2\delta T_b)^2 + (2\delta T_a)^2}$

From error propagation formula,

$$\delta g = \sqrt{(\frac{\partial g}{\partial h_d}\ \delta h_d)^2 + (\frac{\partial g}{\partial T_d}\ \delta T_d)^2} = \sqrt{(\frac{2}{T_d^2}\delta h_d)^2 + (\frac{4h_d}{T_d^3}\delta T_d)^2}$$

Report 'g' as $g_{measured} \pm \delta g$

After obtaining acceleration due to gravity with setup-2, compare it with the same obtained from Section-1 and comment on the results.

Is there any systematic error in determining 'g'? Is it in both the setups or one of them?

# Volume of Metal Bar

Aim: To find volume of the metal bar by using Vernier caliper, Screw gauge and, Travelling microscope.

Apparatus Used:

- ✓ Metal bar
- ✓ Screw gauge
- ✓ Vernier caliper
- ✓ Scale

**Theory:**

## Vernier Caliper:

A device which is used to measure small lengths or distance such as thickness of a metal bar or diameter of cylinder is called Vernier caliper. With the help of Vernier calliper we can calculate correctly up to $1/1000^{th}$ of centimeter or 0.1mm. Vernier caliper can be used to make more precise observations and take more accurate readings than a meter scale. Hence it is preferred over a meter scale. It consists of a main scale along which an auxiliary scale slides known as Vernier scale.

## Working:

To measure the length of an object, it is placed between the two jaws of the Vernier calipers. The distance between the zero of the main scale and the zero of the Vernier scale is equal to length of object. First we find the reading of main scale then find the number of Vernier division which coincide with one of the main scale division. We multiply the vernier divisions by the least count and then add to main scale reading to get the length of an object.

$$\text{Length} = \text{Main scale reading} + \text{least count} \times \text{vernier coincidence}$$

## Least Count:

The least count or the smallest reading which you can get with the instrument, for the scale and it is minimum division and for vernier caliper it can be calculated as;

$$\text{Least count} = \text{one main scale (MS) division} - \text{one Vernier scale (VS) division}$$

or

$$\text{Least Count} = \frac{\text{One Main scale(MS) division}}{\text{Number of divisions in Vernier Scale}}$$

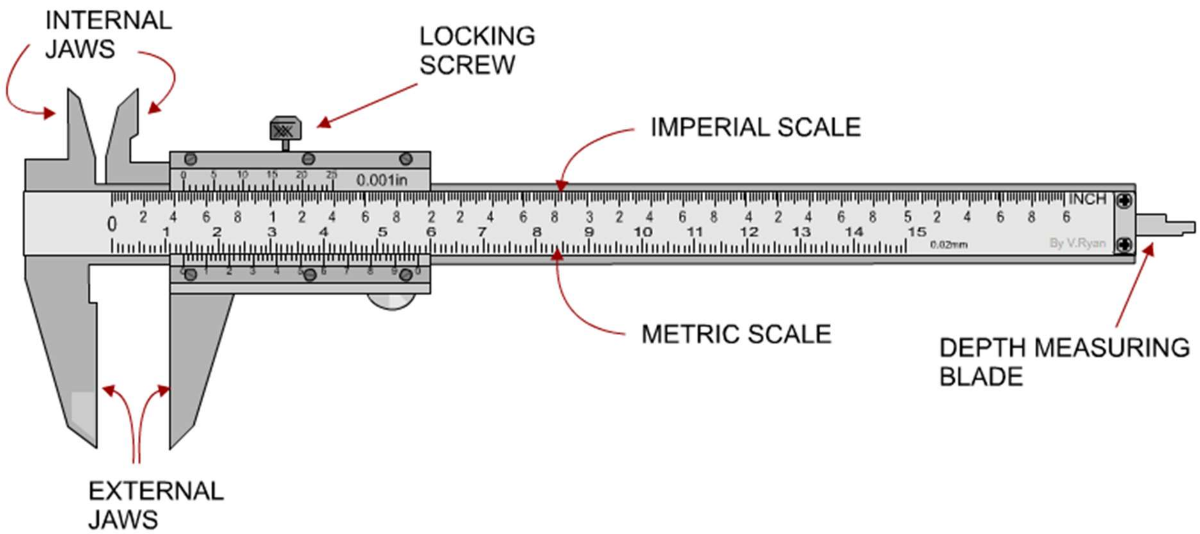


**Figure 1. Vernier Caliper**

**Tabulation 1 for Length of the metal bar using scale:**

| Sl No | Divisions on the scale (N) | Interpolated division of least count (I) | Total (N+I) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| Average length = | | | |

**<u>Screw Gauge:</u>**

The screw gauge is an instrument used for measuring accurately the diameter of a thin wire or the thickness of a sheet of metal. It consists of a U-shaped frame fitted with a screwed spindle which is attached to a thimble.

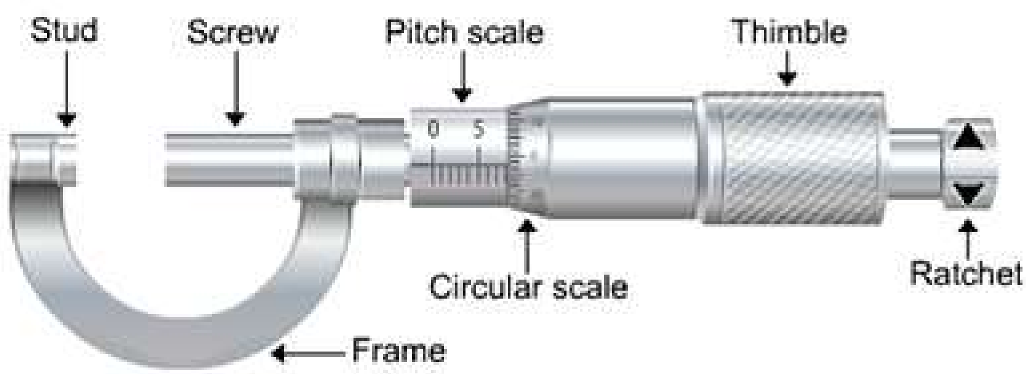


**Figure 2. Screw Gauge**

Parallel to the axis of the thimble, a scale graduated in mm is engraved. This is called pitch scale. A sleeve is attached to the head of the screw.

The head of the screw has a ratchet which avoids undue tightening of the screw. On the thimble there is a circular scale known as head scale which is divided into 50 or 100 equal parts. When the screw is worked, the sleeve moves over the pitch scale.

A stud with a plane end surface called the anvil is fixed on the 'U' frame exactly opposite to the tip of the screw. When the tip of the screw is in contact with the anvil, usually, the zero of the head scale coincides with the zero of the pitch scale.

**Pitch of the Screw Gauge:**

The pitch of the screw is the distance moved by the spindle per revolution. To find this, the distance advanced by the head scale over the pitch scale for a definite number of complete rotation of the screw is determined.

The pitch can be represented as;

$$\text{Pitch of the screw} = \frac{\text{Distance moved by screw}}{\text{No. of full rotations given}} \text{.............(1)}$$

**Least Count of the Screw Gauge:**

The Least count (LC) is the distance moved by the tip of the screw, when the screw is turned through 1 division of the head scale.

The least count can be calculated using the formula;

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}} \text{.............(2)}$$

Then, linear scale reading (L.S.R.) = N

If nth division of circular scale lies over reference line.

Then, circular scale reading (C.S.R.) = n x (L.C.) (L.C. is least count of screw gauge)

Total reading (T.R) = L.S.R. + C.S.R. = N + n x (L.C.).

**Tabulation 2 for thickness of the metallic bar using vernier caliper:**

| Sl. No | Main scale reading (N) | Circular scale division on reference line reading (n) | Circular Scale reading (n x L.C) | Total Reading (N + n x L.C) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| | | | Average value of thickness | |

**Zero Error and Zero Correction**

To get the correct measurement, the zero error must be taken into account. For this purpose, the screw is rotated forward till the screw just touches the anvil and the edge of cap is on the zero mark of the pitch scale. The Screw gauge is held keeping the pitch scale vertical with its zero down wards.

When this is done, anyone of the following three situations can arise:

1. The **zero mark** of the circular scale comes on the reference line. In this case, the zero error and the zero correction, both are nil as shown in fig.3.
2. **Positive zero error:** If zero of the circular scale is below the base line of pitch scale then error is positive zero error or negative correction as shown in fig.3 case-2. In this case, the **zero error is positive** and the **zero correction is negative** depending on how many divisions it is below the base line.
3. **Negative zero error:** When zero of a circular scale is above the base line of the pitch scale then error is negative zero error or positive error as shown in fig. 3 case 3. In this case, the **zero error is negative** and the **zero correction is positive** depending on how many divisions it is above the base line.

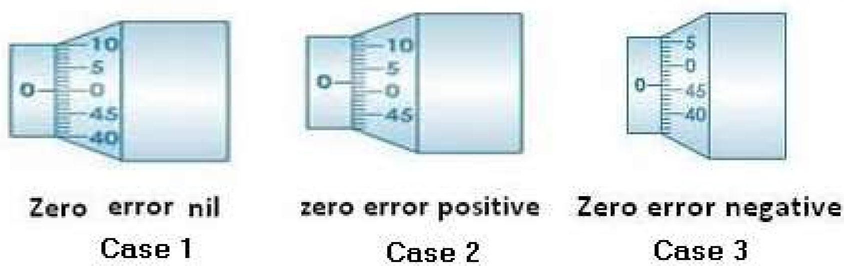


Fig. 3

**Tabulation 3 for measurement of breadth of the metallic bar using screw gauge:**

| Sl No | Main scale reading (N) | Vernier Coincidence ($V_c$) | Vernier scale reading ($V_c$ xL.C) | Total reading (N+$V_c$ x L.C) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| | | | Average value of length | |

Volume of the metal bar, $V = \bar{l} \text{ x } \bar{b} \text{ x } \bar{h}$, where $\bar{l}$, $\bar{b}$and $\bar{h}$ are the average values of length, breadth and height of the metal bar, respectively.

Length is calculated using Vernier caliper, breadth is calculated using travelling microscope and thickness is calculated using screw gauge for better accuracy.

The random error formula for in Volume is given by

$$\delta V = \sqrt{\left(\frac{\partial V}{\partial l}\,\delta l\right)^2 + \left(\frac{\partial V}{\partial b}\,\delta b\right)^2 + \left(\frac{\partial V}{\partial h}\,\delta h\right)^2},$$

$$V = \bar{l} \times \bar{b} \times \bar{h}$$

Where, δl, δb and δh are the errors in length, breadth and thickness, respectively. For scale it is least count/2 and for Vernier caliper and screw gauge is least count.

Hence, the Final Volume can be represented as,

$$V = V_{measured} \pm \delta V$$